

Course ISE 435: Distributed Algorithms in Network
Communication
Recitation 1 Exercise

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1 Exercise 1

Prove the following for undirected graphs:

1. A connected graph on n nodes has at least $n - 1$ edges
2. An acyclic graph on n nodes has at most $n - 1$ edges

1.1 Proof

Assume that $G = (V, E)$ is a graph, N is the number of nodes in G and D is the diameter.

1. Prove by induction on the number of nodes in G .

Base case $|N| = 1$: The minimal size of N is 0 edges, when there are no self loops. $|E| = N - 1$.

Step $N = n \rightarrow n + 1$: Let's take the minimum sized graph of size N which has $N - 1$ edges by the inductive assumption. The new node $N + 1$ must be connected to at least one existing node by an edge to keep the graph connected. That means that $|E_{n+1}| = N = (N + 1) - 1$.

2. Prove by inductions on the number of nodes in G .

Base case $N = 1$: To be acyclic, the graph must have a maximum of 0 edges (or else there is a cycle) $= N - 1 = 0$.

Step $N = n \rightarrow n + 1$: Assume that the graph of size n is maximal and has $n - 1$ edges. Adding one more edge to the n nodes would make a cycle. Adding the node $n + 1$, the graph has no cycles. We can add one more edge, from node $n + 1$ to node n_1 of the of the n nodes without adding a cycle, yielding $n = (n + 1) - 1$ edges. Adding another edge, to some other node n_2 would make a cycle - it's the same as adding an edge from n_1 to n_2 and by the inductive hypothesis that would make the graph cyclic. Therefore we cannot add a second edge and n is the maximum number of edges possible.